

Practice Worksheet for Law of Exponents

Use the laws of exponents you discovered in your investigation to simplify each of the following expressions with positive exponents.

1. $3^3 \bullet 3^4$

11. 3^{-4}

2. $x^8 \bullet x^5$

12. $\frac{8^5}{8^2}$

3. $3x^3 \bullet 4x^2$

13. 27^0

4. $x^2y^4 \bullet x^5y^{12}$

14. $\frac{12x^5}{4x^2}$

5. $(5^2)^3$

15. $\frac{2x^6y^5}{16x^4y}$

6. $(x^4)^5$

16. $\frac{3^5}{3^7}$

7. $(2x)^3$

17. $\frac{4x^5y^2}{20x^3y^4}$

8. $(x^4y^5)^3$

18. $\frac{12xy^2}{3x^4y^2}$

9. $(3x^4y^3z^5)^3$

19. $\left(\frac{2x^4}{3x}\right)^3$

10. $(2x^3)^4(-3x^2y^3)^2$

20. $\frac{18x^{-5}y^4}{12x^{-3}y^{-3}}$

Practice Worksheet for Law of Exponents
Answer Key

Use the laws of exponents you discovered in your investigation to simplify each of the following expressions with positive exponents.

1. $3^3 \bullet 3^4$ 2187

11. 3^{-4} $\frac{1}{81}$

2. $x^8 \bullet x^5$ x^{13}

12. $\frac{8^5}{8^2}$ 512

3. $3x^3 \bullet 4x^2$ $12x^5$

13. 27^0 1

4. $x^2y^4 \bullet x^5y^{12}$ x^7y^{16}

14. $\frac{12x^5}{4x^2}$ $3x^3$

5. $(5^2)^3$ 15,625

15. $\frac{2x^6y^5}{16x^4y}$ $\frac{x^2y^4}{8}$

6. $(x^4)^5$ x^{20}

16. $\frac{3^5}{3^7}$ $\frac{1}{9}$

7. $(2x)^3$ $8x^3$

17. $\frac{4x^5y^2}{20x^3y^4}$ $\frac{x^2}{5y^2}$

8. $(x^4y^5)^3$ $x^{12}y^{15}$

18. $\frac{12xy^2}{3x^4y^2}$ $\frac{4}{x^3}$

9. $(3x^4y^3z^5)^3$ $27x^{12}y^9z^{15}$

19. $\left(\frac{2x^4}{3x}\right)^3$ $\frac{8x^9}{27}$

10. $(2x^3)^4(-3x^2y^3)^2$ $144x^{16}y^6$

20. $\frac{18x^{-5}y^4}{12x^{-3}y^{-3}}$ $\frac{3y^7}{2x^2}$

Investigation of the Laws of Exponents

Step 1—Write each product in expanded form, and then rewrite it in exponential form.

a. $4^2 \bullet 4^3$

b. $10^4 \bullet 10^8$

c. $x^5 \bullet x^{10}$

Step 2—Write a conjecture based upon your results in step 1.

$$x^a \bullet x^b =$$

Step 3—Write the numerator and denominator of each expression in expanded form. Write in simplest form by eliminating common factors and then rewrite the factors that remain in exponential form.

a. $\frac{2^6}{2^4}$

b. $\frac{10^{12}}{10^5}$

c. $\frac{x^6}{x}$

Step 4—Write a conjecture based upon your results in step 3.

$$\frac{x^a}{x^b} =$$

Step 5—Write the numerator and denominator of each expression in expanded form. Write in simplest form by eliminating common factors and then rewrite the factors that remain in exponential form.

a. $\frac{3^2}{3^5}$

b. $\frac{10^4}{10^5}$

c. $\frac{x^2}{x^6}$

Step 6—Rewrite each of the expressions in step 5 using the conjecture you made in step 4.

a. $\frac{3^2}{3^5}$

b. $\frac{10^4}{10^5}$

c. $\frac{x^2}{x^6}$

Step 7—Use your results in step 5 and step 6 to write a conjecture. $\frac{1}{x^a} =$

Step 8—Write the numerator and denominator of each expression in expanded form. Simplify by eliminating common factors and then use the conjecture you made in step 4 to rewrite each of the expressions in exponential form.

a. $\frac{4^5}{4^5} =$

b. $\frac{10^3}{10^3} =$

c. $\frac{x^6}{x^6} =$

Step 9—Write a conjecture based upon what you have discovered in step 8.

Step 10—Write each of the following in expanded form and then simplify by writing it in exponential form.

a. $(2^2)^3$

b. $(5^3)^8$

c. $(x^4)^3$

Step 11—Based upon your results in step 10, can you write a shortcut for taking a power to a power? $(x^a)^b =$

Step 12—Explore each of the following.

a. $(2x)^3$

b. $(3 \bullet 4)^2$

c. $4(2x)^4$

d. $\left(\frac{3}{4}\right)^2$

e. $\left(\frac{x}{y}\right)^4$

Step 13—What statements can you make about your observations in step 12?

Teacher's Notes on the Investigation of the Laws of Exponents

Step 1—Write each product in expanded form, and then rewrite it in exponential form.

$$a. 4^2 \bullet 4^3 = 4 \bullet 4 \bullet 4 \bullet 4 \bullet 4 = 4^5$$

$$b. 10^4 \bullet 10^8 = 10 \bullet 10 \bullet 10 \bullet 10 \bullet 10 \bullet 10 \bullet 10 \bullet 10 \bullet 10 \bullet 10 \bullet 10 \bullet 10 \bullet 10 \bullet 10 = 10^{12}$$

$$c. x^5 \bullet x^{10} = x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x = x^{15}$$

Step 2—Write a conjecture based upon your results in step 1.

$$x^a \bullet x^b = x^{a+b}$$

Step 3—Write the numerator and denominator of each expression in expanded form. Write in simplest form by eliminating common factors and then rewrite the factors that remain in exponential form.

$$a. \frac{2^6}{2^4} = \frac{2 \bullet 2 \bullet 2 \bullet 2 \bullet 2 \bullet 2}{2 \bullet 2 \bullet 2 \bullet 2} = 2^2$$

$$b. \frac{10^{12}}{10^5} = \frac{10 \bullet 10 \bullet 10 \bullet 10 \bullet 10 \bullet 10 \bullet 10 \bullet 10 \bullet 10 \bullet 10 \bullet 10 \bullet 10}{10 \bullet 10 \bullet 10 \bullet 10 \bullet 10} = 10^7$$

$$c. \frac{x^6}{x} = \frac{x \bullet x \bullet x \bullet x \bullet x \bullet x}{x} = x^5$$

Step 4—Write a conjecture based upon your results in step 3.

$$\frac{x^a}{x^b} = x^{a-b}$$

Step 5—Write the numerator and denominator of each expression in expanded form. Write in simplest form by eliminating common factors and then rewrite the factors that remain in exponential form.

$$a. \frac{3^2}{3^5} = \frac{3 \bullet 3}{3 \bullet 3 \bullet 3 \bullet 3 \bullet 3} = \frac{1}{3^3}$$

$$b. \frac{10^4}{10^5} = \frac{10 \bullet 10 \bullet 10 \bullet 10}{10 \bullet 10 \bullet 10 \bullet 10 \bullet 10} = \frac{1}{10}$$

$$c. \frac{x^2}{x^6} = \frac{x \bullet x}{x \bullet x \bullet x \bullet x \bullet x \bullet x} = \frac{1}{x^4}$$

Step 6—Rewrite each of the expressions in step 5 using the conjecture you made in step 4.

a. $\frac{3^2}{3^5} = 3^{2-5} = 3^{-3}$

b. $\frac{10^4}{10^5} = 10^{4-5} = 10^{-1}$

c. $\frac{x^2}{x^6} = x^{2-6} = x^{-4}$

Step 7—Use your results in step 5 and step 6 to write a conjecture. $\frac{1}{x^a} = x^{-a}$

Step 8—Write the numerator and denominator of each expression in expanded form. Simplify by eliminating common factors and then use the conjecture you made in step 4 to rewrite each of the expressions.

a. $\frac{4^5}{4^5} = \frac{4 \bullet 4 \bullet 4 \bullet 4 \bullet 4}{4 \bullet 4 \bullet 4 \bullet 4 \bullet 4} = 1$ $4^{5-5} = 4^0$

b. $\frac{10^3}{10^3} = \frac{10 \bullet 10 \bullet 10}{10 \bullet 10 \bullet 10} = 1$ $10^{3-3} = 10^0$

c. $\frac{x^6}{x^6} = \frac{x \bullet x \bullet x \bullet x \bullet x \bullet x}{x \bullet x \bullet x \bullet x \bullet x \bullet x} = 1$ $x^{6-6} = x^0$

Step 9—Write a conjecture based upon what you have discovered in step 8.

$$x^0 = 1$$

Step 10—Write each of the following in expanded form and simplify by writing it exponential form.

$$\text{a. } (2^2)^3 = 2^2 \cdot 2^2 \cdot 2^2 = 2 \cdot 2^2 \cdot 2^2 \cdot 2^2 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^6$$

$$\text{b. } (5^3)^8 = 5^3 \cdot 5^3 \cdot 5^3 \cdot 5^3 \cdot 5^3 \cdot 5^3 \cdot 5^3 \cdot 5^3 = 5^{24}$$

$$\text{c. } (x^4)^3 = x^4 \cdot x^4 \cdot x^4 = x^{12}$$

Step 11—Based upon your results in step 10, can you write a shortcut for taking a power to a power? $(x^a)^b = x^{ab}$

Step 12—Explore each of the following.

$$\text{a. } (2x)^3 = 2x \cdot 2x \cdot 2x = 8x^3$$

$$\text{b. } (3 \cdot 4)^2 = 12^2 = 144$$

$$\text{c. } 4(2x)^4 = 4(2x \cdot 2x \cdot 2x \cdot 2x) = 64x^4$$

$$\text{d. } \left(\frac{3}{4}\right)^2 = \frac{3 \cdot 3}{4 \cdot 4} = \frac{9}{16}$$

$$\text{e. } \left(\frac{x}{y}\right)^4 = \frac{x \cdot x \cdot x \cdot x}{y \cdot y \cdot y \cdot y} = \frac{x^4}{y^4}$$

Step 13—What statements can you make about your observations in step 12?

$$(xy)^a = x^a y^a \quad \left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$$

Everything within the parentheses must be taken to the given power

Summarize/Debrief

- Select groups to share their results. (Assign each group a conjecture to present—make sure you watch for different approaches to the investigation so different methods may be presented.)

1. Product Law of Exponents: $x^a \bullet x^b = x^{a+b}$

2. Quotient Law of Exponents: $\frac{x^a}{x^b} = x^{a-b}$

3. Definition of Negative Exponents: $x^{-a} = \frac{1}{x^a}$ ($x \neq 0$)

4. Zero Exponent: $x^0 = 1$ ($x \neq 0$) 0^0 is undefined

5. Power of a Power: $(x^a)^b = x^{ab}$

6. Power of a Product: $(xy)^a = x^a y^a$

7. Power of a Quotient: $\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$ ($y \neq 0$)